FIRM AND WORKER DYNAMICS IN AN AGING LABOR MARKET

Niklas Engbom
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Technology and Aging Workforce Conference
**Figure I: Share of labor force aged 40+ & dynamism**

- Fraction of labor force aged 40 and older

- Log dynamism (normalized in 1986)

Figure I: Share of labor force aged 40+ & dynamism

Background

Figure I: Share of labor force aged 40+ & dynamism

Aging Firm dynamics Worker dynamics Growth Longer time series Shift-share
What is the impact of aging on dynamism?

1. Reduced-form empirical assessment

   - Exploit predictable variation in aging across US states
1. **Reduced-form empirical assessment**

- Exploit predictable variation in aging across US states
- Aging predicts *40–50% of declines* & negative growth effect
What is the impact of aging on dynamism?

1. **Reduced-form empirical assessment**
   - Exploit predictable variation in aging across US states
   - Aging predicts 40–50% of declines & negative growth effect

2. **Structural assessment**
   - Theory that links firm dyn., worker dyn. & growth to aging
1. Reduced-form empirical assessment
   ▪ Exploit predictable variation in aging across US states
   ▪ Aging predicts 40–50% of declines & negative growth effect

2. Structural assessment
   ▪ Theory that links firm dyn., worker dyn. & growth to aging
   ▪ 40–50% of declines in firm & worker dynamism and $-\frac{1}{4}$ percentage point in annual economic growth
What is the impact of aging on dynamism?

1. Reduced-form empirical assessment
   - Exploit predictable variation in aging across US states
   - Aging predicts 40–50% of declines & negative growth effect

2. Structural assessment
   - Theory that links firm dyn., worker dyn. & growth to aging
   - 40–50% of declines in firm & worker dynamism and $-\frac{1}{4}$ percentage point in annual economic growth
   - Half due to equilibrium effects
Cross-state Evidence on the Impact of Aging
State-fixed effect framework

- State-year data on dynamism & age composition 1978–2014

- Regress dynamism on share 40–64, controlling for state + year

\[
\log (y_{st}) = \alpha \log (\text{share}_{st}^{40-64}) + \xi_s + \xi_t + X_{st} \beta + \varepsilon_{st}
\]

- Standard errors clustered at state and year
State-year data on dynamism & age composition 1978–2014

Regress dynamism on share 40–64, controlling for state + year

\[ \log (y_{st}^a) = \alpha \log (\text{share}_{st}^{40–64}) + \xi_s + \xi_t + \mathbf{X}_{st} \beta + \xi_a + \varepsilon_{st}^a \]

Standard errors clustered at state and year
Identifying variation

- Variation in timing & magnitude of aging across states

Figure II: Fraction aged 40–64 in four selected states
**Endogenous Mobility**

**Issue:** Mobility across states in response to dynamism
**Endogenous mobility**

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- Differential mobility in response to **temporary** variation
Endogenous Mobility

**Issue**: Mobility across states in response to dynamism

- Differential mobility in response to *temporary* variation

**Instrument current age composition with**

1. 10-year lagged age composition
   - Only effect on dynamism through current age composition
   - Strong explanatory power on current age composition
Endogenous Mobility

**Issue:** Mobility across states in response to dynamism

- **Differential** mobility in response to *temporary* variation

**Instrument current age composition with**

1. 10-year lagged age composition
   - Only effect on dynamism through current age composition
   - Strong explanatory power on current age composition

2. Birth rates 40–64 years earlier
   - Only effect on dynamism through current age composition
   - Decent explanatory power on current age composition
## Firm Dynamism

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## Worker Dynamism and Growth

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<th>Panel D: Growth in GDP per worker</th>
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Predicted impact of aging

(A) Turnover rate

(B) Entry rate

(C) EU hazard

(D) UE hazard

Raw long series
A Job Ladder Model with Creative Destruction
Key ingredients

- **Job ladder**: Ranking of firms that workers gradually climb

- **Entrepreneurial choice**

- **Creative destruction**: Entrants push out incumbents
Demographics & preferences

○ **Agents**: Unit mass of individuals, $a = 1, \ldots, A$
  - Move to the next age at rate $\kappa(a)$
  - Oldest age group dies at rate $\kappa(A)$ and is replaced by offspring

○ **Preferences**: Risk-neutral and altruistic w.r.t. offspring

$$
\mathbb{E}_t \int_t^{\infty} \exp\left(-\tilde{\rho}(\tau - t)\right) \left[ C(\tau) + \tilde{B}(\tau) \right] d\tau
$$

where $\tilde{B}(\tau) = B(\tau)$ if unemployed; zero o.w.
○ **Multiworker firms**: Idiosyncratic productivity $\tilde{z}$

$$d\tilde{z}(t) = \mu_o dt + \sigma dW(t)$$

○ **Production**: At match level, $y(z, x) = e^{\tilde{z}} \times x$

  ○ $x = $ quality of match; starts at $x = 1$

  ○ Jumps to $x_b$ or $x_g$, $x_b < 1 < x_g$, with equal prob at rate $\psi$

  ○ Worker flows $>>$ job flows
Growth & stationarity

**Two sources of growth:**

1. Growth of incumbents at exogenous rate $\mu_o$

2. Selection of firms at endogenous rate $\mu$

    $\implies$ Total growth rate $\mu_e = \mu_o + \mu$

**Transformation:** $z = \tilde{z} - \tilde{z}(t)$ etc.

- Incumbents fall behind at rate of obsolescence, $\mu = \mu_e - \mu_o$
**INDIVIDUAL’S PROBLEM**

**WHEN TO SWITCH EMPLOYER & BECOME ENTREPRENEUR**

- Job finding rate $\lambda$ from both U & E

- Entrepreneurship opportunities at rate $\gamma$
  - Entry cost $c \sim \Omega(a)$ and has to quit job (if employed)
  - Draws productivity from innovation distribution $\phi(z)$
  - Sells idea to MF and returns to labor market as unemployed

- Wage setting following Cahuc et al (2006)
Firm’s problem

Post vacancies $v$ subject to cost $C(v) = r + c(v)$

- $c(v)$ is strictly convex flow cost per vacancy
- $r$ is fixed cost associated with employing a unit of capital

$\implies$ Stop paying $\Rightarrow$ exit

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) + \right. \right.$$ 

$$\left. + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right] - c(v) \right\} - r \underbrace{- \mu J'(z)}_{\text{drift in } z} + \frac{\sigma^2}{2} \underbrace{J''(z)}_{\text{shocks to } z}$$

- **Vacancy policy**: $v(p)$ defined by FOC
Closing the model

**Households own firms through mutual fund**

- Avoids age of founder as state (Romer, 1990)
- Rents out $K$ capital to firms in competitive market
  \[ \implies \text{Factor in fixed supply} \Rightarrow \text{Creative destruction} \]

**Labor market:** Cobb-Douglas matching function, \( m = \chi V^\alpha \)
CHARACTERIZING BEHAVIOR & THE EQUILIBRIUM

**Prop. 1 (Mismatch and worker dynamism)**

(a) Better matched individuals are less likely to move

(b) A better matched labor market discourages vacancy creation

**Prop. 2 (Mismatch and entrepreneurship)**

(a) Better matched individuals are less entrepreneurial

(b) A better matched labor market discourages entrepreneurship

**Prop. 3 (Amplification)**

Rate of obsolescence increases in the aggregate entry rate, \( \mu = \frac{e}{\zeta} \)

Less entry \( \Rightarrow \) Lower rate of obsol. \( \Rightarrow \) Less mismatch \( \Rightarrow \) Less entry
Amplification

Aging

↓

Entry

Worker pairings with firms

↑

Growth

Michael Bloomberg
Aging

Aging and entry

Michael Bloomberg  Aging and JJ  Aging and entry
**Amplification**

- **Aging**

- **Worker**

- **Pairing with firms**
Amplification

Aging  ↓  Entry

Worker
↓
Pairing
↑
With firms

Michael Bloomberg  Aging and JJ  Aging and entry
Amplification

Aging \rightarrow \downarrow \text{Entry}

Worker pairing with firms

↑ Pairing with firms
Aging
↓
Entry

Worker pairing with firms

↑

Growth
Amplification

Aging

Worker pairing with firms

→ ↓ Entry

↑

Growth
Structural estimate of the impact of aging

Aging explains

1. 40–50% of declines in entry, exit, incumbent job reallocation, EU and JJ mobility; modest fall in UE

2. $-\frac{1}{4}$ percentage points decline in growth

3. Half due to equilibrium effects

Policy: Regulation/taxation or immigration?
1. Aging typically accounts for at most half of declines
   - Labor supply (Karahan et al, 2016)
   - Licensing (Kleiner and Krueger, 2013)
   - Training requirements (Cairo, 2013)
   - EPL (Autor et al., 2007)

2. Anecdotal evidence that aging has reduced dynamism & growth in other countries
   - A rigorous cross-country analysis is missing
Thank you
Appendix A
Figure IV: Share 40 and older and EU hazard

Fraction of labor force aged 40 and older

Monthly worker hazard

Share older

EU hazard
**Age Composition**

(A) Age distribution (labor force)

(B) Share 40+ (labor force)

(C) Age distribution (working age)

(D) Share 40+ (working age)

Motivation
Firm dynamics definitions

Data

- Annual data on firms and establishments covering private sector

Definitions

- Job creation: \( JC_t = \sum_i (size_{it} - size_{it-1})^+ \)
- Job destruction: \( JD_t = \sum_i (- (size_{it} - size_{it-1}))^+ \)

\[
JC_t + JD_t = J_{inc}^t + J_{dec}^t + J_{entry}^t + J_{exit}^t
\]

Job reallocation\( _t = \) Inc job reallocation\( _t + \) Estabs. turnover\( _t \)
**Firm Dynamics**

**Figure VI: Establishment reallocation rates**

![Graph showing establishment reallocation rates from 1978 to 2013.](image)

- **Motivation**: Exit/entry, By firm age, By industry.
ENTRY AND EXIT

(A) Establishment

(B) Establishment (unweighted)

(C) Firm

(D) Firm (unweighted)
Dynamics by firm age

(A) Establishment exit rate

(B) Incumbent job reallocation

(C) Firm exit rate

Appendix  Karahan et al  Motivation
Dynamics by industry

(A) Turnover

(B) Job reallocation

(C) Turnover

(D) Job reallocation

Appendix  Motivation
Worker mobility definitions

Data

○ SIPP (1984–2013)
○ CPS (1978–2015)
○ BLS (1948–2015)

Definitions

○ $EU_{it} = \text{employed in month } t, \text{ unemployed in } t + 1$
○ $UE_{it} = \text{unemployed in } t, \text{ employed in } t + 1$
○ $JJ_{it} = \text{employed in } t, \text{ different main employer in } t + 1$
Worker dynamics

- Large fall in EU & JJ hazard
- Little evidence of secular decline in UE hazard

(A) EU hazard

(B) JJ hazard

(C) UE hazard

SIPP, CPS, BLS

Motivation
LN AND NL FLOWS

- Declines in the hazard of moving in and out of the labor force

(A) LN HAZARD

(B) NL HAZARD
Annual growth in real GDP per worker slowed from 2.6% in 1984–1988 to 1.7% in 2012–2016

Figure XII: Annual HP-filtered growth rate
Karahan, Pugsley and Sahin (2016)

- Labor supply growth explains $\frac{1}{4}$ of fall in start-up rate
- No change in incumbent life-cycle dynamics

Two key differences

1. "Quality"/composition of labor force rather than quantity
2. Partly different set of outcomes: Worker dynamics, incumbent dynamics and growth
Denote by $rate_{late}^{a}$ age-conditional mobility rates in 2012–2014.

Denote by $share_{p}^{a}$ the share of the labor force in age $a$ in period $p$.

Direct effect = change due to shift in age composition under fixed age-conditional mobility rates.

$$rate_{direct}^{a} = \sum_{a} rate_{late}^{a} [share_{a}^{early} - share_{a}^{late}]$$
### Shift-share

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<th>(2) Late</th>
<th>(3) % change</th>
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<td></td>
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<td><strong>Panel A: JJ mobility</strong></td>
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Michael Bloomberg

- Partner at Solomon Brothers, laid off in 1981 (at age 39)
- Started financial service company Bloomberg LP
- Current net worth: $47.8bn

Would he have started Bloomberg if he had not been laid off?

- Walt Disney, JK Rowling, Thomas Edison, Mark Cuban, Oprah Winfrey, Sallie Krawcheck, Bernie Marcus and Arthur Blank...
**Entrepreneurship entry by age**

**Figure XIII: Entrepreneurship entry by age**

- **Probability of starting a business** by age group:
  - 18-24
  - 25-34
  - 35-44
  - 45-54
  - 55+

- **Age categories**:
  - Baseline
  - Opportunistic
  - Expects to grow

**Model Calibration**
**Post entry performance by age of founder**

(A) Cover owners’ salary

(B) Hired at least one employee

(C) Share surviving

(D) Log firm size
Individuals may be either employed or unemployed
○ Individuals may be either employed or unemployed

○ Search with the same efficiency (normalized to one)
Search and Matching

- Individuals may be either employed or unemployed
- Search with the same efficiency (normalized to one)
- If firms post $\bar{v}$ vacancies, total number of matches equals $\chi \bar{v}^\alpha$
○ Individuals may be either employed or unemployed

○ Search with the same efficiency (normalized to one)

○ If firms post $\bar{v}$ vacancies, total number of matches equals $\chi \bar{v}^\alpha$

○ Denote by $\lambda$ rate at which individuals meet with open vacancies, $q$ rate at which vacancy contacts individuals

$$
\lambda = \chi \bar{v}^\alpha, \quad q = \chi \bar{v}^{\alpha-1}
$$
Wage setting


- **Unemployed**: Outside value plus $\beta$ of surplus

- **Employed (I)**: Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus $\beta$ of differential

- **Employed (II)**: Poacher with higher valuation
  - Switch to poacher, get current match plus $\beta$ of differential

$\Rightarrow$ Renegotiation when one party has credible threat
Wage setting


- **UNEMPLOYED**: Outside value plus $\beta$ of surplus

- **EMPLOYED (I)**: Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus $\beta$ of differential

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Renegotiation when one party has credible threat

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**Wage setting**

**Offer matching framework of Cahuc et al (2006)**

- **Unemployed**: Outside value plus $\beta$ of surplus

- **Employed (I)**: Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus $\beta$ of differential

- **Employed (II)**: Poacher with higher valuation
  - Switch to poacher, get current match plus $\beta$ of differential

$\Rightarrow$ **Renegotiation when one party has credible threat**
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by

$$z = \log\left(\frac{Z(t)}{Z(0)}\right) \quad \text{normalized log firm productivity}$$

$$r \quad \text{the normalized price of a marketing specialist}$$

$$\phi(z) \quad \text{the normalized innovation distribution}$$

$$\rho = \tilde{\rho} - \mu \quad \text{the effective discount rate} =$$

- Incumbent firm productivity drifts at $-\mu$ while $r$ is constant.
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by
  1. $z = \log(Z(t)/\underline{Z(t)})$ normalized log firm productivity.
Balanced growth

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- Normalize by $Z(t)$ and denote by
  1. $z = \log(Z(t)/\underline{Z}(t))$ normalized log firm productivity
  2. $r$ the normalized price of a marketing specialist
Balanced growth

- On the BGP, \( Z(t) \) and \( \tilde{r}(t) \) grow at endogenous rate \( \mu \), while incumbent firm productivity in expectation does not change.

- Study transformed economy in which \( Z(t) \) and \( \tilde{r}(t) \) do not grow.

- Normalize by \( Z(t) \) and denote by
  1. \( z = \log(Z(t)/\bar{Z}(t)) \) normalized log firm productivity
  2. \( r \) the normalized price of a marketing specialist
  3. \( \phi(z) \) the normalized innovation distribution
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by
  1. $z = \log(Z(t)/\bar{Z}(t))$ normalized log firm productivity
  2. $r$ the normalized price of a marketing specialist
  3. $\phi(z)$ the normalized innovation distribution
  4. $\rho = \tilde{\rho} - \mu$ the effective discount rate
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by
  
  1. $z = \log(Z(t)/Z(t))$ normalized log firm productivity
  2. $r$ the normalized price of a marketing specialist
  3. $\phi(z)$ the normalized innovation distribution
  4. $\rho = \tilde{\rho} - \mu$ the effective discount rate

$\implies$ Incumbent firm productivity drifts at $-\mu$ while $r$ is constant.
Value of unemployment

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \]

Aging
Value of unemployment

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_{0}^{\infty} \{V(z, x, a) - U(a)\}^+ dF(z) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
Value of unemployment

\[
\rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z)
\]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
Value of unemployment

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)

Aging + Job offer

Entrepreneurship opportunity
\[
\rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z) + \gamma(a) \int_{\bar{c}} \{E - c\}^+ d\Omega(c)
\]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)
- Opportunity to start business at rate \( \gamma(a) \)
\[ \rho U(a) = b + \kappa(a)[U(a + 1) - U(a)] + \lambda \beta \int_{0}^{\infty} \{V(z, x_u, a) - U(a)\}^+ dF(z) + \gamma(a) \int_{c}^{\tilde{c}} \{E - c\}^+ d\Omega(c) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)

- Opportunity to start business at rate \( \gamma(a) \)
  - Associated entry cost \( c \) drawn from \( \Omega \)
  - \( E \) denotes expected value of entrepreneurship
**Value of unemployment**

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z) + \gamma(a) \int_{\bar{c}}^c \{E - c\}^+ d\Omega(c) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)

- Opportunity to start business at rate \( \gamma(a) \)
  - Associated entry cost \( c \) drawn from \( \Omega \)
  - \( E \) denotes expected value of entrepreneurship

- **Decision rules:** \( z_u(x_u, a) \)
**Value of unemployment**

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z) + \gamma(a) \int_{\tilde{c}}^{\infty} \{E - c\}^+ d\Omega(c) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)

- Opportunity to start business at rate \( \gamma(a) \)
  - Associated entry cost \( c \) drawn from \( \Omega \)
  - \( E \) denotes expected value of entrepreneurship

- **Decision rules:** \( z_u(x_u, a) \) and \( \bar{c}_u \)
Value of match with known quality

\[ \rho V (z, x, a) = e^z - \mu \frac{\partial V (z, x, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V (z, x, a)}{\partial z^2} + \]

- drift in \( z \)
- shocks to \( z \)

\[ + \kappa (a) [\max \{ V (z, x, a + 1) , U(a + 1) \} - V (z, x, a)] + \]

- individual ages

\[ + \lambda \beta \int_{0}^{\infty} \max \{ V (z', x_u, a) - V (z, x, a) , 0 \} dF(z') + \]

- new job offer

\[ + \gamma (a) \int_{\bar{c}}^{c} \max \{ E - c - V (z, x, a) + U(a), 0 \} d\Omega(c) + \]

- entrepreneurship opportunity
An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$. 

Back to main
An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$

She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea
An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$.

She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea.

Hence the expected value of entry equals

$$E = \int_{0}^{\infty} J(z) d\Phi(z)$$
Denote by \( J(z) \) the value of hiring to a firm

\[
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+ \right) \right] + \right. \\
+ (1 - u) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a) \right\} - c(v) - \left. \frac{r}{2} J''(z) \right. \\
- \left. \mu J'(z) \right. + \left. \frac{\sigma^2}{2} J''(z) \right. .
\]
Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v \geq 0} \left\{ v (1 - \beta) q \left[ \sum_a \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) + \right. \right.$$  

$$
\left. + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right\} - c(v) \right\} - r \text{ fixed cost} - \mu J'(z) + \frac{\sigma^2}{2} J''(z)$$

Post vacancies $v$ subject to $c(v)$
Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_{a} \left( u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+ \right) + \right. \right.$$

$$+ (1 - u) \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a) \right\} - c(v) \left. \right\} - r \text{ fixed cost - } \mu J'(z) + \frac{\sigma^2}{2} J''(z)$$

Post vacancies $v$ subject to $c(v)$

Has to pay fixed cost $r$ to remain in hiring market
**Value of hiring to firm**

- Denote by $J(z)$ the value of hiring to a firm

$$
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) \right] + \right.
$$

$$
+ (1 - u) \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a) \right\} - c(v) - \frac{r}{2} J'(z) + \frac{\sigma^2}{2} J''(z)
$$

- Post vacancies $v$ subject to $c(v)$

- Has to pay fixed cost $r$ to remain in hiring market

- **Decision rules:** vacancy policy, $v(z)$, and exit threshold, $z$
Vacancy policy

\[ v(z) = \left\{ \frac{(1 - \beta)q}{c_v} \left[ \sum_a u(a) \{V(z, x_u, a) - U(a)\}^+ + (1 - u) \int [V(z, x_u, a) - V(z', x, a)]^+ dG(z', x, a) \right] \right\}^{1/\eta} \]

Individual is unemployed

Individual is employed

1. Larger share of individuals are employed
2. Employed individuals are less mismatched

Less labor market mismatch \( \Rightarrow \) less vacancy creation
Vacancy policy

\[ v(z) = \left\{ \frac{(1 - \beta)q}{c_v} \left[ \sum_a u(a) \{ V(z, x_u, a) - U(a) \}^+ + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ \, dG(z', x, a) \right] \right\}^{1/\eta} \]

Less labor market mismatch \( \implies \) less vacancy creation

1. Larger share of individuals are employed
Vacancy policy

\[ v(z) = \left\{ \begin{array}{l}
\frac{(1 - \beta)q}{c_v} \left[ \sum_{a} u(a) \{ V(z, x_u, a) - U(a) \}^+ + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right] \\
\text{Individual is unemployed} & \text{Individual is employed}
\end{array} \right\}^{1/\eta} \]

Less labor market mismatch \implies less vacancy creation

1. Larger share of individuals are employed

2. Employed individuals are less mismatched
Value functions \( \{U, V, E, J\} \); policies \( \{\bar{c}_u, z_u(x, a), z(z, x, a), \bar{c}(z, x, a)\} \); policies \( \{z, v(z)\} \); numbers \( \{r, e, \mu, \bar{v}, \lambda, q\} \); and distributions \( \{h(z), f(z), u(a), g(z, x, a)\} \); such that

1. Value and policy functions of unemployed, match and recruiting firm solve the respective problems

2. The aggregate entry rate \( e \) is consistent with individual behavior

3. The growth rate \( \mu \) is consistent with the entry rate

4. Aggregate vacancies \( \bar{v} \) are consistent with firm behavior and the finding rates are \( \lambda = \chi \bar{v}^\alpha \), \( q = \chi \bar{v}^{\alpha - 1} \)

5. Distributions solve respective KFE and are stationary
1. $h(z)$ denotes the pdf of recruiting firms

2. $f(z)$ denotes the \textit{vacancy-weighted} pdf of recruiting firms

3. $u(a)$ denotes the mass of unemployed individuals of age $a$

4. $g(z, x, a)$ denotes the pdf of employed individuals

For all densities, upper case letters denote the corresponding cdf
The distribution of recruiting firms, $h$, solves the KFE

$$0 = \mu h'(z) + \frac{\sigma^2}{2} h''(z) + e\zeta \exp(-\zeta z), \quad z > 0$$  \hspace{1cm} (1)

subject to,

$$h(0) = 0, \quad \int_{0}^{\infty} h(z) \, dz = 1, \quad e = \frac{\sigma^2}{2} h'(0)$$ \hspace{1cm} (2)

where $e$ is the aggregate entry rate

- Last condition can be seen by integrating (1) from 0 to $\infty$, which gives $0 = -\mu h(0) - \sigma^2 / 2h'(0) + e$, and imposing $h(0) = 0$
(1) is a second-order ordinary differential equation with solution,

\[ h(z) = \frac{e}{\mu - \frac{\sigma^2}{2}\zeta} \left[ \exp(-\zeta z) - \exp\left(-\frac{2\mu}{\sigma^2} z\right) \right] \]
(1) is a second-order ordinary differential equation with solution,

\[ h(z) = \frac{e}{\mu - \frac{\sigma^2}{2} \zeta} \left[ \exp(-\zeta z) - \exp\left(-\frac{2\mu}{\sigma^2} z\right) \right] \tag{3} \]

where the growth rate of the economy is a function of the aggregate entry rate of entrepreneurs,

\[ \mu = \frac{e}{\zeta} \tag{4} \]
The vacancy-weighted distribution of firms, \( f(z) \), equals the density of recruiting firms at \( z \) times the amount of vacancies they post,

\[
f(z) = \frac{v(z)h(z)}{\bar{v}}
\]

where \( v(z) \) is the firm’s optimal vacancy policy and

\[
\bar{v} = \int_{0}^{\infty} v(\tilde{z}) dh(\tilde{z})
\]
On the BGP, \( g(z, x, a) \) satisfies the KFE

\[
0 = \mu \frac{\partial g(z, x, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 g(z, x, a)}{\partial z^2} + \lambda \frac{u(a)}{1 - u} f(z) \mathbf{1} \{ x = x_u \} \mathbf{1} \{ z > z^u(x_u, a) \} + \text{inflow from unemployment}
\]

\[
+ \kappa(a - 1) \mathbf{1} \{ z > z^u(x_u, a) \} g(z, x, a - 1) - \kappa(a) g(z, x, a) + \text{inflow from aging}
\]

\[
+ \lambda f(z) \mathbf{1} \{ x = x_u \} \int \mathbf{1} \{ z > z^e(z', x', a) \} G(dz', dx', a) - \lambda \left[ 1 - F(z^e(z, x, a)) \right] g(z, x, a) + \text{inflow to higher rungs in job ladder}
\]

\[
+ \psi \mathbf{1} \{ z > z^u(x, a) \} \pi(x) g(z, x_u, a) - \psi \mathbf{1} \{ x = x_u \} g(z, x, a) - \gamma(a) g(z, x, a) \Omega \left( c^e(z, x, a) \right) \text{outflow from learning}
\]

with \( \pi(x_u) = 0 \) and \( g(z, x, 0) \equiv 0, \forall z, x \), subject to workers exiting at the boundary so that the density is zero and the pdf integrates to one.
The mass of unemployed of each age group, \( u(a) \), satisfies,

\[
0 = -\lambda \left[ 1 - F \left( \tilde{z}^u(x_u, a) \right) \right] u(a) + \left( 1 - u(a) \right) \sum_x \frac{\sigma^2}{2} \frac{\partial g(\tilde{z}^u(x, a), x, a)}{\partial z} + \]

\[
\text{outflow to employment} \quad \text{individuals drifting below the threshold}
\]

\[
+ (1 - u(a)) \psi \pi(x_b) G \left( \tilde{z}^u(x_b, a), x_u, a \right) + 1 \left\{ a = 1 \right\} \kappa(A) - \]

\[
\text{individuals jumping below the threshold due to learning} \quad \text{newborn}
\]

\[
- \kappa(a) u(a) + \kappa(a - 1) \left[ u(a - 1) + (1 - u) \sum_x G \left( \tilde{z}^u(x, a), x, a - 1 \right) \right] + \]

\[
\text{outflow from aging} \quad \text{inflow from aging}
\]

\[
+ (1 - u(a)) \gamma(a) \int \Omega \left( \tilde{c}^e(z, x, a) \right) G(dz, dx, a)
\]

\[
\text{entry to entrepreneurship}
\]

with the convention that \( u(0) = 0 \)
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int \left[ 1 - F(z^e(z, x, a)) \right] dG(z, x, a) \]

(8)
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] \, dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_{a} m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F(z^e(z, x, a))] \, d\hat{G}(z, x|a) \quad (8) \]
4 Effects of Aging on JJ Hazard

\[ JJ = \lambda \int \left[ 1 - F \left( z^e(z, x, a) \right) \right] dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int \left[ 1 - F \left( z^e(z, x, a) \right) \right] d\hat{G}(z, x|a) \quad (8) \]

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]  

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F (z^e(z, x, a))] dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F (z^e(z, x, a))] d\hat{G}(z, x|a) \quad (8) \]

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation

3. \( F \) may change as firms change their vacancy posting decisions
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F (z^e(z, x, a))] \, dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F (z^e(z, x, a))] \, d\hat{G}(z, x|a) \quad (8) \]

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation

3. \( F \) may change as firms change their vacancy posting decisions

4. Aging may give rise to changes in age-conditional labor market mismatch, \( \hat{G}(z, x|a) \)
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

(9)
3 EFFECTS OF AGING ON ENTRY RATE

\[
e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\tilde{c}^e (z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\tilde{c}^u) \sum_a u(a) \gamma(a) \right\}
\]

\[
e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\tilde{c}^e (z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\tilde{c}^u) \right\}
\]

(9)
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) \, dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] \, d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \quad (9) \]

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter
3 Effects of Aging on Entry Rate

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \tag{9} \]

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect \( \bar{c}^e(z, x, a) \) (\( \bar{c}^u \)) as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]  \hspace{1cm} (9)

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect \( \bar{c}^e(z, x, a) \) (\( \bar{c}^u \)) as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting

3. Age conditional labor market mismatch—\( \hat{G}(z, x|a) \) and \( u(a)/m(a) \)—may change through equilibrium effects
Appendix D
**Strategy**

**Target**: Salient features of aggregate firm & worker dynamism in BDS + SIPP in 2012–2014

- Monthly frequency
- 3 age groups
- Pre-set a few parameters to standard values
- Remaining parameters internally
## Calibrated Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Labor market mobility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_v$ Cost of vacancy creation</td>
<td>Aggregate UE (2005–07)</td>
<td>$4.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\pi(x_b)$ $P$ (match is low productive)</td>
<td>Aggregate EU</td>
<td>0.5</td>
</tr>
<tr>
<td>$x_g$ Productivity of high prod. match</td>
<td>Aggregate JJ</td>
<td>1.3</td>
</tr>
<tr>
<td>$\psi$ Rate of learning</td>
<td>Timing of decline in JJ with tenure</td>
<td>0.043</td>
</tr>
<tr>
<td>$b$ Flow value of unemployment</td>
<td>Indifference at margin</td>
<td>1.09</td>
</tr>
<tr>
<td><strong>Panel B: Entrepreneurship</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$ Innovation distribution</td>
<td>Growth due to selection</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma(a)$ Entrepreneurship opportunity</td>
<td>Entry rate and entry rate by age</td>
<td>$[4.2; 4.5; 2.1] \times 10^{-3}$</td>
</tr>
<tr>
<td>$C$ Dispersion in entry cost</td>
<td>Decline in entry with tenure</td>
<td>72</td>
</tr>
<tr>
<td><strong>Panel C: Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$ Curvature of vacancy creation</td>
<td>Size distribution of entrants</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma$ Shocks to productivity</td>
<td>Size distribution</td>
<td>$7 \times 10^{-3}$</td>
</tr>
<tr>
<td>$d$ Exit shock for firms</td>
<td>Average exit rate</td>
<td>$3.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K$ Capital</td>
<td>Average firm size</td>
<td>0.13</td>
</tr>
</tbody>
</table>
### Calibrated values

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<tbody>
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<td><strong>Panel A: Labor market mobility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_v$</td>
<td>Cost of vacancy creation</td>
<td>Aggregate UE (2005–07)</td>
</tr>
<tr>
<td>$\pi(x_b)$</td>
<td>$P($match is low productive$)$</td>
<td>Aggregate EU</td>
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<tr>
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<td>Productivity of high prod. match</td>
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</tr>
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<td>Indifference at margin</td>
</tr>
<tr>
<td><strong>Panel B: Entrepreneurship</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Innovation distribution</td>
<td>Growth due to selection</td>
</tr>
<tr>
<td>$\gamma(a)$</td>
<td>Entrepreneurship opportunity</td>
<td>Entry rate and entry rate by age</td>
</tr>
<tr>
<td>$C$</td>
<td>Dispersion in entry cost</td>
<td>Decline in entry with tenure</td>
</tr>
<tr>
<td><strong>Panel C: Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>Curvature of vacancy creation</td>
<td>Size distribution of entrants</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Shocks to productivity</td>
<td>Size distribution</td>
</tr>
<tr>
<td>$d$</td>
<td>Exit shock for firms</td>
<td>Average exit rate</td>
</tr>
<tr>
<td>$K$</td>
<td>Capital</td>
<td>Average firm size</td>
</tr>
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</table>
Calibrated values

- $C \Rightarrow$ Elasticity of entry to net value

Figure XV: Tenure profile of entrepreneurship entry hazard
## Calibrated values

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<tr>
<td>$\pi(x_b)$ $P$(match is low productive)</td>
<td>Aggregate EU</td>
<td>0.5</td>
</tr>
<tr>
<td>$x_g$ Productivity of high prod. match</td>
<td>Aggregate JJ</td>
<td>1.3</td>
</tr>
<tr>
<td>$\psi$ Rate of learning</td>
<td>Timing of decline in JJ with tenure</td>
<td>0.043</td>
</tr>
<tr>
<td>$b$ Flow value of unemployment</td>
<td>Indifference at margin</td>
<td>1.09</td>
</tr>
<tr>
<td><strong>Panel B: Entrepreneurship</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$ Innovation distribution</td>
<td>Growth due to selection</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma(a)$ Entrepreneurship opportunity</td>
<td>Entry rate and entry rate by age</td>
<td>[4.2; 4.5; 2.1]$ \times 10^{-3}$</td>
</tr>
<tr>
<td>$C$ Dispersion in entry cost</td>
<td>Decline in entry with tenure</td>
<td>72</td>
</tr>
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<td></td>
<td></td>
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<td>2</td>
</tr>
<tr>
<td>$\sigma$ Shocks to productivity</td>
<td>Size distribution</td>
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Calibrated values

- $\eta \Rightarrow$ Elasticity of vacancy creation to net value

(A) Employment share by size, entrants

(B) Employment share by size

- Firm size: 1-249, 250-499, 500-999, 1000+
- Employment share: 0, 0.25, 0.5, 0.75, 1
# Calibrated Values

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</tr>
<tr>
<td>$K$    Capital</td>
<td>Average firm size</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Success I: Life cycle firm dynamics

- Calibration targets aggregate firm size and exit rate

[Diagram of firm size, exit rate, and incumbent job reallocation]
Success II: Life cycle labor market mobility

- Calibration targets aggregate JJ & EU hazard

⇒ SUPPORTS JOB LADDER AND LEARNING MECHANISMS

(A) EU

(B) JJ

<table>
<thead>
<tr>
<th>Age</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35-44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45+</td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Monthly hazard rate</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td></td>
</tr>
<tr>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>.015</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Monthly hazard rate</th>
<th>Target</th>
</tr>
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<tbody>
<tr>
<td>.02</td>
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</tr>
<tr>
<td>.04</td>
<td></td>
</tr>
<tr>
<td>.06</td>
<td></td>
</tr>
</tbody>
</table>

UE by age  Wage by tenure  Back
Success III: Linking firms and workers

- Matches hire & separation rates as function of firm growth

⇒ Supports joint model of firm & worker dynamics

(A) Data

(B) Model

Hires
Separations
Job creation/destruction

Hires
Separations
Job creation/destruction
### Table I: Pre-set parameter values

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ Discount rate</td>
<td>Annual interest rate of 4%</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\chi$ Matching efficiency</td>
<td>Normalization</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha$ Elasticity of matching function</td>
<td>Petrongolo and Pissarides (2001)</td>
<td>0.7</td>
</tr>
<tr>
<td>$\beta$ Bargaining power</td>
<td>Bagger et al (2014)</td>
<td>0.3</td>
</tr>
</tbody>
</table>
## Calibration targets—individuals

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Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\implies \uparrow$ EU hazard

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Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\implies \uparrow$ EU hazard

- $\uparrow x_g \implies \uparrow$ opportunity cost of JJ mobility $\implies \downarrow$ JJ hazard

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Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\implies \uparrow$ EU hazard

- $\uparrow x_g \implies \uparrow$ opportunity cost of JJ mobility $\implies \downarrow$ JJ hazard

- $\uparrow \psi \implies$ learning is faster $\implies$ JJ falls quickly with tenure

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</table>
### Calibration targets—entrepreneurs

The table below summarizes the calibration targets for entrepreneurs.

| Description                          | Target                                | Value                              |
|--------------------------------------|---------------------------------------|.....................................|
| \(\gamma(a)\) Entrepreneurship opportunity | Entry rate by age                      | \([4.2; 4.5; 2.1] \times 10^{-3}\) |
| \(\zeta\) Innovation distribution     | Growth due to selection                | 20                                 |
| \(C\) Dispersion in entry cost       | Decline in entry with tenure          | 72                                 |

Opportunity cost is positively correlated with tenure and hence, the decline in entry with tenure informs the change in the value of entry for a given change in the entry rate by age.
# Calibration targets—entrepreneurs

\[ \Omega \sim U(-\Delta, \Delta) \]

\[ \Delta \uparrow \Rightarrow \Delta \downarrow \text{change in entry for given change in value of entry} \]

Opportunity cost is positively correlated with tenure and hence decline in entry with tenure informs:

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<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( \gamma(a) )</td>
<td>Entrepreneurship opportunity</td>
<td>Entry rate by age</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Innovation distribution</td>
<td>Growth due to selection</td>
</tr>
<tr>
<td>( C )</td>
<td>Dispersion in entry cost</td>
<td>Decline in entry with tenure</td>
</tr>
</tbody>
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Calibration targets—entrepreneurs

- $\Omega \sim U(-C, C)$

- $\uparrow C \implies \downarrow$ change in entry for given change in value of entry

- Opportunity cost is positively correlated with tenure and hence decline in entry with tenure informs $C$

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Calibration targets—firms

- $\eta \uparrow \Rightarrow$ more costly to hire many workers $\Rightarrow$ less dispersion in initial firm size

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<tr>
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<td>$\sigma$ Shocks to productivity</td>
<td>Size distribution</td>
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<tr>
<td>$K$ Capital</td>
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</tr>
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Calibration targets—firms

- $\eta \uparrow \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size

- Introduce small probability of firm death, $d$, that is independent of firm productivity

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**Calibration targets—firms**

- \( \eta \uparrow \Rightarrow \text{more costly to hire many workers} \Rightarrow \text{less dispersion in initial firm size} \)

- Introduce small probability of firm death, \( d \), that is independent of firm productivity

- \( \sigma \uparrow \Rightarrow \text{dispersion in steady-state firm productivity} \Rightarrow \text{dispersion in steady-state firm size} \)

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Figure XVIII: Worker mobility by tenure

(A) JJ

(B) EU
Figure XX: UE hazard by age
Model matches well average wages by tenure $\Rightarrow$ confidence in $\beta$
Figure XXII: Average wage by firm age
Exit rate by firm size

Figure XXIII: Exit rate by firm size
FIGURE XXIV: EMPLOYMENT SHARES

Employment share by firm age

Model | Data
--- | ---
0.25 | 0.5
0.75 | 1.0
1.25 | 1.5
1.75 | 2.0
2.25 | 2.5
2.75 | 3.0
3.25 | 3.5
Average wage by firm size

Figure XXV: Average wage by firm size
HIRES AND SEPARATIONS BY ORIGIN AND DESTINATION

(A) Hires by origin

(B) Separations by destination

(C) Share of hires from other firms

(D) Net poaching
Figure XXVII: Average worker age by firm age
Appendix E
Details on change in age composition

Change the age composition of the economy to 1986 and evaluate its impact on dynamism

- Increase the rate at which older individuals exit the market, $\kappa(3)$
  1. Increases the share of young people
  2. Shortens the time individuals expect to remain in the market

- The retirement age has not changed suggesting that individuals did not expect to spend less time in the market in the 1980s

⇒ Use original $\kappa(3)$ in value functions and new $\kappa(3)$ when computing individual transitions
Target change in share of older $\Rightarrow$ Understates somewhat fall in the share of young

**Table II:** Share of individuals in each age group by period

<table>
<thead>
<tr>
<th>Age Group</th>
<th>(1) Early</th>
<th>(2) Model</th>
<th>(3) Late</th>
<th>(4) Model</th>
<th>(5) Change</th>
<th>(6) Model</th>
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<tbody>
<tr>
<td>Young</td>
<td>0.492</td>
<td>0.434</td>
<td>0.356</td>
<td>0.339</td>
<td>-0.136</td>
<td>-0.095</td>
</tr>
<tr>
<td>Middle aged</td>
<td>0.231</td>
<td>0.289</td>
<td>0.208</td>
<td>0.226</td>
<td>-0.023</td>
<td>-0.063</td>
</tr>
<tr>
<td>Older</td>
<td>0.277</td>
<td>0.277</td>
<td>0.436</td>
<td>0.436</td>
<td>0.159</td>
<td>0.158</td>
</tr>
</tbody>
</table>

*Note: Empirical moments corresponds to the share of the labor force age 16–34 (young), 35–44 (middle aged) and 45+ (older) in 1986 and 2015 from the BLS.*
JJ VERSUS UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out
     $\Rightarrow$ Only modest decline in $\lambda$

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity
     $\Rightarrow$ Less likely individual accepts job offer
     $\Rightarrow$ JJ hazard falls over and above the decline in $\lambda$
JJ VERSUS UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
Two opposing effects on vacancy creation

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In contrast, the less dynamic economy implies that

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⇒ Less likely individual accepts job offer
⇒ JJ hazard falls over and above the decline in $\lambda$
JJ versus UE

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Two opposing effects on vacancy creation

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out
    \[ \Rightarrow \text{Only modest decline in } \lambda \]

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity
    \[ \Rightarrow \text{Less likely individual accepts job offer} \]

\[ \Rightarrow \text{JJ hazard falls over and above the decline in } \lambda \]
Figure XXVIII: Change in vacancy policy and firm distribution

(A) Vacancy creation

(B) Firm productivity distribution

- Percent change in jobs created
- Log firm productivity
- Fraction of firms
- Log firm productivity
- Early period
- Late period
The decline in the passthrough

Decker et al. (2017)

1. The fall in job reallocation is not due to a more benign economic environment

2. Older firms adjust employment less in response to productivity shocks

3. Employment has shifted towards older firms, accounting for some of the decline in the passthrough

4. The response has fallen within firm age groups
1. No change in variance of shocks

2. Lower passthrough of older firms as equilibrium outcome
   - Employment change to productivity shock is linked to \# ranks
   - Log distance between ranks is larger further up the ladder
   - Shock moves firm fewer ranks at top $\Rightarrow$ smaller employment response
   - Older, surviving firms are on average further up the ladder

3. Aging results in shift of employment towards older firms

4. Employment has also shifted up the ladder within age groups
### Table III: Passthrough from productivity to employment innovations

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All firms</td>
<td>Young firms</td>
<td>Mature firms</td>
</tr>
<tr>
<td>( \Delta \text{TFP} )</td>
<td>3.504***</td>
<td>5.604***</td>
<td>2.394***</td>
</tr>
<tr>
<td>Late period ( \times \Delta \text{TFP} )</td>
<td>-0.566***</td>
<td>-0.212***</td>
<td>-0.177***</td>
</tr>
</tbody>
</table>

Note: Young firms are <5 years, mature firms \( \geq 5 \) years. Outcome variable is annual change in log firm size. Independent variable is annual change in log firm productivity. Weighted by employment.

\[ \implies \text{Declines driven by weaker passthrough} \]
**Table IV: Decomposition of change in log output**

<table>
<thead>
<tr>
<th>(1) Age composition</th>
<th>(2) Firm productivity</th>
<th>(3) Match productivity</th>
<th>(4) Net output</th>
<th>(5) Discounted net output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014</td>
<td>0.044</td>
<td>0.004</td>
<td>0.055</td>
<td>-0.040</td>
</tr>
</tbody>
</table>
## Table IV: Decomposition of change in log output

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<tr>
<th>(1) Age composition</th>
<th>(2) Firm productivity</th>
<th>(3) Match productivity</th>
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<td>0.044</td>
<td>0.004</td>
<td><strong>0.055</strong></td>
<td>-0.040</td>
</tr>
</tbody>
</table>
## Decomposition of level difference

### Table IV: Decomposition of change in log output

<table>
<thead>
<tr>
<th></th>
<th>(1) Age composition</th>
<th>(2) Firm productivity</th>
<th>(3) Match productivity</th>
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<th>(5) Discounted net output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.014</td>
<td>0.044</td>
<td>0.004</td>
<td>0.055</td>
<td>-0.040</td>
</tr>
</tbody>
</table>
\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]
Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]

Table V: Decomposing the change in the JJ and entry hazard

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entry hazard</strong></td>
<td>% change</td>
<td>% of total</td>
<td>% change</td>
<td>% of total</td>
</tr>
<tr>
<td>Direct: m(a)</td>
<td><strong>10.5</strong></td>
<td><strong>47.5</strong></td>
<td><strong>7.0</strong></td>
<td><strong>53.6</strong></td>
</tr>
</tbody>
</table>
Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

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</tr>
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<tbody>
<tr>
<td>Entry hazard</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% change</td>
<td>10.5</td>
<td>47.5</td>
<td>7.0</td>
<td>53.6</td>
</tr>
<tr>
<td>% of total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JJ hazard</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% change</td>
<td></td>
<td></td>
<td>23.3</td>
<td>179</td>
</tr>
<tr>
<td>% of total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Direct: \( m(a) \)
Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega [\tilde{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\tilde{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (\tilde{z}^e(z, x, a))] d\hat{G}(z, x|a) \]

**Table V: Decomposing the change in the JJ and entry hazard**

<table>
<thead>
<tr>
<th></th>
<th>(1) % change</th>
<th>(2) % of total</th>
<th>(3) % change</th>
<th>(4) % of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct: ( m(a) )</td>
<td>10.5</td>
<td>47.5</td>
<td>7.0</td>
<td>53.6</td>
</tr>
<tr>
<td>Policy: ( \tilde{c}^e(z, x, a) / \lambda [1 - F (\tilde{z}^e(z, x, a))] )</td>
<td>1.2</td>
<td>5.4</td>
<td>-17.3</td>
<td>-133</td>
</tr>
</tbody>
</table>
\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega [\tilde{c}^e(z, x, a)] \, d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\tilde{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (\tilde{z}^e(z, x, a))] \, d\hat{G}(z, x|a) \]

**Table V: Decomposing the change in the JJ and entry hazard**

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<tr>
<td></td>
<td>Entry hazard</td>
<td></td>
<td>JJ hazard</td>
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</tr>
<tr>
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<td>1.2</td>
<td>5.4</td>
<td><strong>-17.3</strong></td>
<td>-133</td>
</tr>
</tbody>
</table>

\( \hat{c} \) and \( \hat{G} \) are terms involving integrals and hazards related to the change in entry and the final cost.
Decomposition

\[
e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\tilde{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\tilde{c}^u) \right\}
\]

\[
JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int \left[ 1 - F (z^e(z, x, a)) \right] d\hat{G}(z, x|a)
\]

Table V: Decomposing the change in the JJ and entry hazard

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</tr>
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</tr>
<tr>
<td>Direct: (m(a))</td>
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</tr>
<tr>
<td>Policy: (\tilde{c}^e(z, x, a)/\lambda [1 - F (z^e(z, x, a))])</td>
<td>1.2</td>
</tr>
<tr>
<td>Mismatch: (\hat{G}(z, x</td>
<td>a))</td>
</tr>
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Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega \left[ \bar{c}^e(z, x, a) \right] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]

**Table V: Decomposing the change in the JJ and entry hazard**

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<td>5.4</td>
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<td>-133</td>
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<tr>
<td>Mismatch: ( \hat{G}(z, x</td>
<td>a) )</td>
<td>10.4</td>
<td>47.2</td>
<td><strong>23.3</strong></td>
</tr>
</tbody>
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\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] \, d\hat{G}(z, x|a) \]

Table V: Decomposing the change in the JJ and entry hazard

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<th>(3) % change</th>
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<td></td>
<td></td>
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<td>5.4</td>
<td>-17.3</td>
<td>-133</td>
</tr>
<tr>
<td>Mismatch: ( \hat{G}(z, x</td>
<td>a) )</td>
<td>10.4</td>
<td>47.2</td>
<td>23.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>22.2</td>
<td>100</td>
<td>13.1</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure XXX: Share with high match productivity

Shift in match productivity
How much does JJ fall with mismatch?

Figure XXXI: Distribution of older individuals and JJ hazard
What moments of the data inform the effects

Figure XXXII: Tenure profile of JJ mobility

Large equilibrium effects are not hardwired
Figure XXXIII: Tenure distribution

Tenure distribution

Data, late  Data, early  Model, late  Model, early
Aging explains key changes in life-cycle firm dynamics

- Employment has shifted substantially towards older firms
- Exit has fallen the most for old firms
- Age conditional firm size has declined

(A) Employment share

(B) Exit rate

(C) Average firm size
- Aging in model replicates patterns across states
- Relatively larger effect on mobility rates late in careers
### Shift-share analysis

#### Table VI: Shift share analysis with firm and worker age

<table>
<thead>
<tr>
<th></th>
<th>(1) Data</th>
<th>(2) Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Firm dynamics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exit</td>
<td>-0.008</td>
<td>-0.003</td>
</tr>
<tr>
<td><em>Direct effect</em></td>
<td>-0.008</td>
<td>-0.004</td>
</tr>
<tr>
<td><em>% of total</em></td>
<td>96.8</td>
<td>142.4</td>
</tr>
<tr>
<td>Incumbent</td>
<td>-0.045</td>
<td>-0.024</td>
</tr>
<tr>
<td><em>Direct effect</em></td>
<td>-0.010</td>
<td>-0.018</td>
</tr>
<tr>
<td><em>% of total</em></td>
<td>22.7</td>
<td>74.4</td>
</tr>
</tbody>
</table>

*Panel B: Worker dynamics*

<table>
<thead>
<tr>
<th></th>
<th>(1) Data</th>
<th>(2) Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU</td>
<td>-0.003</td>
<td>-0.001</td>
</tr>
<tr>
<td><em>Direct effect</em></td>
<td>-0.001</td>
<td>-0.000</td>
</tr>
<tr>
<td><em>% of total</em></td>
<td>20.7</td>
<td>34.4</td>
</tr>
<tr>
<td>JJ</td>
<td>-0.005</td>
<td>-0.002</td>
</tr>
<tr>
<td><em>Direct effect</em></td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td><em>% of total</em></td>
<td>40.8</td>
<td>51.7</td>
</tr>
</tbody>
</table>
Aging generates modest shift of employment to larger firms in line with the data over this period.
No aging of potential hires

\[
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( \tilde{m}(a) \frac{u(a)}{m(a)} \{ V(z, x_u, a) - U(a) \}^+ \right) + \right. \right. \\
\left. \left. + \sum_a \left\{ \tilde{m}(a) \left( 1 - \frac{u(a)}{m(a)} \right) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ d\hat{G}(z', x|a) \right\} \right] - c(v) \right\} - \right. \\
- r - \mu J'(z) + \frac{\sigma^2}{2} J''(z)
\]

- Hold firms’ expectations of age composition fixed at original age composition, \( \tilde{m}(a) \)

\[\implies \text{No change in age-composition externality} \]
Table VII: No direct congestion externality due to aging

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No aging of hires</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Share</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel A: Firm dynamics**

| Entry rate             | -0.012 | -0.008 | 72    |
| Job reallocation       | -0.039 | -0.031 | 80    |

**Panel B: Worker dynamics**

| EU hazard              | -0.001 | -0.001 | 87    |
| JJ hazard              | -0.002 | -0.002 | 72    |

**Panel C: Growth**

| Growth per worker      | -0.26  | -0.18  | 69    |
- Adjust $\gamma(a)$ to have no direct effect through aging entrepreneurs

### Table VIII: No aging of potential entrepreneurs

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>No aging of entrep.</td>
<td>Share</td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: Firm dynamics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry rate</td>
<td>-0.012</td>
<td>-0.003</td>
<td>27</td>
</tr>
<tr>
<td>Job reallocation</td>
<td>-0.039</td>
<td>-0.009</td>
<td>22</td>
</tr>
<tr>
<td><strong>Panel B: Worker dynamics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU hazard</td>
<td>-0.001</td>
<td>-0.001</td>
<td>61</td>
</tr>
<tr>
<td>JJ hazard</td>
<td>-0.002</td>
<td>-0.002</td>
<td>65</td>
</tr>
<tr>
<td><strong>Panel C: Growth</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth per worker</td>
<td>-0.26</td>
<td>-0.11</td>
<td>42</td>
</tr>
</tbody>
</table>
Transition dynamics

- Start with 1986 BGP
- Adjust $\kappa(3)$ and decision rules to 2014 BGP starting in 1990
- Relatively fast convergence of entry rate
- Level effect outweighs growth effect initially

(A) Share older

(B) Entry rate

(C) Output per worker
Discussion of transition

- Would want to eventually solve for full transition path

- Difficulty is that sequence of distributions $G(z, x, a; t)$ becomes a state

- Well known issue in search models—cannot boil down problem to shooting only an interest rate or average wage
# Inequality and Income Dynamics

## Panel A: Inequality

<table>
<thead>
<tr>
<th></th>
<th>Young Data</th>
<th>Young Model</th>
<th>Old Data</th>
<th>Old Model</th>
<th>Change Data</th>
<th>Change Model</th>
<th>Change Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. of productivity</td>
<td>0.35</td>
<td>0.13</td>
<td>0.42</td>
<td>0.14</td>
<td>0.07</td>
<td>0.01</td>
<td>14</td>
</tr>
<tr>
<td>Variance of firm pay</td>
<td>0.40</td>
<td>0.45</td>
<td>0.48</td>
<td>0.46</td>
<td>0.08</td>
<td>0.02</td>
<td>21</td>
</tr>
</tbody>
</table>

## Panel B: Annual Income Innovations

<table>
<thead>
<tr>
<th></th>
<th>Young Data</th>
<th>Young Model</th>
<th>Old Data</th>
<th>Old Model</th>
<th>Change Data</th>
<th>Change Model</th>
<th>Change Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. of innovations</td>
<td>0.55</td>
<td>0.54</td>
<td>0.51</td>
<td>0.52</td>
<td>-0.04</td>
<td>-0.02</td>
<td>62</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.21</td>
<td>-0.25</td>
<td>-0.31</td>
<td>-0.32</td>
<td>-0.10</td>
<td>-0.07</td>
<td>71</td>
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</tbody>
</table>
2nd and 3rd Moments of Income Innovations

(A) Standard deviation

(B) Skewness
Appendix F
Data

- Demographic data from the March CPS and Census Bureau’s Intercensal Censi projections
- Establishment and firm dynamics from the BDS
- Merged CPS monthly files for worker mobility rates
- State real GDP per worker from state private sector GDP (BEA), regional CPIs (BLS), and private sector employment (BDS)
Four other prominent changes

1. Increasing gender and racial diversity
   - Estimated coefficients on share female and non-white are in most cases not statistically significant
   - Typically predict a small *increase* in dynamism
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3. Slowdown in labor supply growth
   - Confirming Karahan et al. (2016), labor supply growth is positively correlated with entry
   - But does not alter conclusion regarding the importance of the age composition
Increasing diversity and education

Figure XXXIX: Share female, non-white and with a college degree

(A) Diversity

(B) College
Labor supply growth

(A) $\Delta$ labor force

(B) $\Delta$ working age population

(C) Entry (lf)

(D) Entry (wp)

- Labor force
- Working age population
- Age composition
- Actual
- Labor supply growth